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## ANALYTICAL INVESTIGATION OF PERIODIC MEDIA WITH NEGATIVE PARAMETERS

K. Vytovtov

Radiophysics Department, Dnepropetrovsk National University, Dnepropetrovsk-50, Ukraine  
49050, e-mail: vytovtov@mailru.com

### ABSTRACT

The electromagnetic properties of an artificial layered material with negative permittivity and permeability are studied analytically. To analyse the translation matrix method is employed. The translation matrix for multilayered structure is found in analytical form. The reflection and propagation coefficients of multilayered plate are written in analytical form. The propagation conditions of a wave within periodic layered structure with negative parameters are obtained in analytical form.

### INTRODUCTION

Materials with negative permittivity and permeability are very interesting at last time for theoretical and experimental investigation. The wave behavior within homogeneous material has been described in [1] at first. The experimental research proofing of existence of such media has been presented in [2]. The possible kinds of different material with negative parameters have been described in [2,3,4]. Probable applications of such material have been considered in [5]. Possibility of having negative permittivity and permeability in omega media for certain range of frequencies has been shown in [6].

However the periodic structures with negative parameters have not been considered. The problem of propagation and reflection by an inhomogeneous (layered) plate is also interesting for our consideration.

### STATEMENT OF THE PROBLEM

The layered medium with negative parameters  $\mu$  and  $\varepsilon$  of the layers is considered in this work. Our purpose is investigation of the propagation conditions of the wave within an infinite periodic layered medium with arbitrary number of layers and the propagation and reflection coefficients of a multilayered plate. The translation matrix method is used for analysis. The translation matrix relates the field components and derivations at end of the period to these at beginning of this period. The propagation conditions will have been found in the analytical form also. Using the translation matrix the expressions of the propagation and reflection coefficients will have been written in analytical form.

### TRANSLATION MATRIX

Translation matrix of a layer with negative permittivity and permeability is

$$\mathbf{M}_i = \begin{vmatrix} \cos k_i z & j k_i \sin k_i z \\ \frac{j}{k_i} \sin k_i z & \cos k_i z \end{vmatrix}$$

The translation matrix for a multilayered structure is found as production of matrices of the layers and after mathematical transformations we can write

$$\mathbf{L}(\lambda) = \sum_{q=1}^{2^{N-1}} \left\{ \frac{1}{2^{N-1}} f_{q,N} \sqrt{k_N/k_1} \times e^{\sum_{i=1}^{N-1} \text{Ln} \left( 1 + \frac{k_{i+1} f_{q,i+1}}{k_i f_{q,i}} \right)} \times \mathbf{L}_q \right\},$$

where

$$\mathbf{L}_q = \begin{vmatrix} \sqrt{\frac{k_1}{k_N}} \cos \left[ \sum_{i=1}^N (\varphi_i f_{q,i}) \right] & -j \sqrt{k_1 k_N} \sin \left[ \sum_{i=1}^N (\varphi_i f_{q,i}) \right] \\ -j \sqrt{k_1 k_N} \sin \left[ \sum_{i=1}^N (\varphi_i f_{q,i}) \right] & \sqrt{\frac{k_N}{k_1}} \cos \left[ \sum_{i=1}^N (\varphi_i f_{q,i}) \right] \end{vmatrix},$$

$$f_{q,i} = \text{sign} \left\{ \sin \left[ \frac{\pi}{2^{N+1-i}} (2q-1) \right] \right\}.$$

is the optical thickness of the  $i$ -th wave.  $f_{q,i}$  is the function introduced in [7].

According with the stable theory the propagation conditions in an infinite medium is determined by the expression  $\text{tr} \mathbf{L} = \pm 2$ . Thus for the considered case we have

$$\left| \sum_{q=1}^{2^{N-1}} \frac{1}{2^{N-1}} f_{q,N} \left( \frac{k_N}{k_1} + \frac{k_1}{k_N} \right) \times e^{\sum_{i=1}^{N-1} \text{Ln} \left( 1 + \frac{k_{i+1} f_{q,i+1}}{k_i f_{q,i}} \right)} \cos \left[ \sum_{i=1}^N (\varphi_i f_{q,i}) \right] \right| = 2.$$

## PROPAGATION AND REFLECTION COEFFICIENTS

The propagation and reflection coefficients of multilayered plate are written in the form analogous to ones for an isotropic layered plate

$$R = \frac{(M_{11} + M_{12}\sigma_2)\sigma_1 - (M_{21} + M_{22}\sigma_2)}{(M_{11} + M_{12}\sigma_2)\sigma_1 + (M_{21} + M_{22}\sigma_2)} \quad T = \frac{2\sigma_1}{(M_{11} + M_{12}\sigma_2)\sigma_1 + (M_{21} + M_{22}\sigma_2)}$$

Here  $\sigma_{1,2}$  is the wavenumbers in first and second medium accordingly,  $M_{kl}$  is the element of the translation matrix.

## NUMERICAL EXAMPLE

For example the two-layered medium with the parameters  $\varepsilon_1 = -1.1$   $\varepsilon_2 = 12$   $\mu_1 = -1$   $\mu_2 = -1$   $d_1 = 7 \times 10^{-6}$   $d_2 = 0.1$  is considered. Wave propagation normal to the interfaces is studied.

The dependence of the module of the eigennumber of the translation matrix on frequency is shown in Fig.1 and Fig.2. As it is seen that in low frequency region there are propagation regions, but in high-frequency region there only points of propagation instead regions.

The dependence of the reflection coefficient on frequency is presented in Fig.3. In region to 5 GHz the reflection coefficient has resonances. The physical reason of this is the fact that the wave is not extinguished completely in the layer with negative parameters.

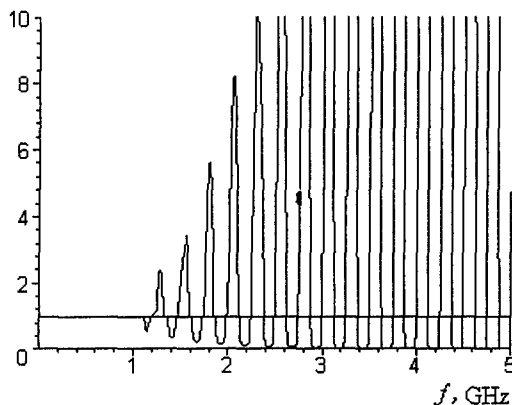


Fig.1. The dependence of the eigennumber of the translation matrix on frequency

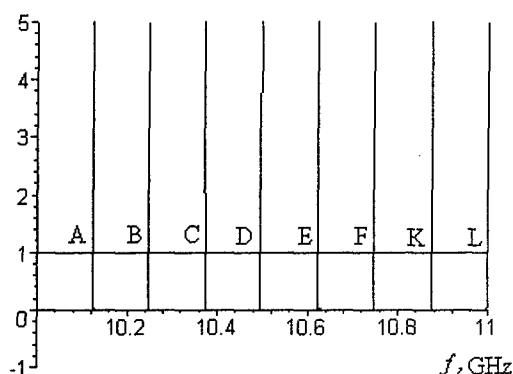


Fig.2. The dependence of the eigennumber of the translation matrix on frequency

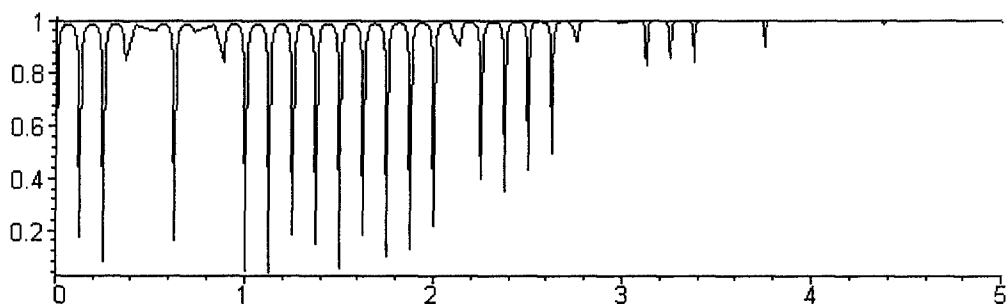


Fig.3. The dependence of the reflection coefficient on frequency

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